Opportunities to Promote Mathematical Content Knowledge for Primary Teaching

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Understanding the development of pre-service teachers' mathematical content knowledge (MCK) is important for improving primary mathematics' teacher education. This paper reports on a case study, Rose and her opportunities to develop MCK during the four years of her program. Program opportunities to promote MCK when planning and practicing primary teaching included: coursework experiences and responding to assessment requirements. Discussion includes the Knowledge Quartet: *foundation knowledge*, *transformation*, *connection* and *contingency*. By fourth-year, Rose demonstrated development of different categories of MCK during practice teaching.

Teaching primary students, pre-service teachers, practicing teachers and working with university colleagues has shaped our understanding of what is needed to transform primary mathematics for the 21st century. There is consensus that teachers require content knowledge in the core subject of mathematics to a level of the students they teach (e.g. Australian Association of Mathematics Teachers [AAMT], 2006; Australian Institute for Teaching and School Leadership (AITSL), 2012; Shulman, 1986). A review of the literature confirmed that pre-service teachers' understanding of MCK was of concern and a topic of interest nationally and internationally (e.g., Callingham, Beswick, Chick, Clark, et al., 2011; Tatto, Schwille, Senk, Ingvarson, et al., 2012).

An effective teacher demonstrates more than common content knowledge (CCK) and relies on many other important categories of subject matter knowledge (Ball, Thames, Phelps, 2008) or foundation knowledge, as well as beliefs about mathematics and pedagogical content knowledge (PCK) (Rowland, Turner, Thwaites, & Huckstep, 2009). This paper reports on the development of one primary, pre-service teacher's MCK (Rose), including the different categories of MCK she demonstrated during her practice teaching experiences and the different program situations or opportunities to learn MCK that influenced her development of knowledge for teaching (e.g. Ball et al., 2008; Rowland et al., 2009).

Literature Review

MCK is an important knowledge required when learning to teach (Carre & Ernest, 1993; Reynolds, 1992). Shulman (1986) explained content knowledge as concerned with expertise in the particular discipline being taught. Further studies have extended Shulman's notion of content knowledge (e.g. Ball et al., 2008; Ma, 1999; Rowland et al., 2009). Pre-service teachers can often underestimate the importance of MCK because of their beliefs (Ambrose, 2004). The findings of Australian studies have identified weaknesses in pre-service teachers' MCK with many relying on procedural methods (Goos, Smith, & Thornton, 2008). Similarly, Cooney and Wiegel's (2003) detailed review of literature examining the mathematics in mathematics teacher education, highlighted worldwide concern regarding pre-service teachers' lack of understanding of mathematics. The

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knowledge that prospective teachers bring from their school experiences are often rule-based (Ball, 1990). Deciding on ways to measure and analyse the knowledge teachers use in mathematics is important in developing an understanding of how mathematical knowledge affects student learning (Ball, Bass, & Hill, 2004).

Goos et al., (2008) completed a MERGA four-year review of research relating to pre-service education of teachers of mathematics, reporting that there has been an increase in and extensive research on, pre-service teacher knowledge. They reported a lack of research on the development of pre-service teachers' mathematical knowledge over time. Clement's (2008) overview noted that much of the research conducted between 2004 and 2007 drew on responses from questionnaires. He expressed concern that when responding to questionnaires, the participants may respond with answers they felt the researcher wanted and that multiple data collection methods would have strengthened these studies.

From a research perspective, frameworks of teacher knowledge can assist with deepening understanding of the different categories used to describe this knowledge, as well as its use for effective teaching of mathematics (Bobis, Higgins, Cavanagh, & Roche, 2012). The *Knowledge Quartet* framework has been used when observing beginning, primary mathematics teachers (Rowland et al., 2009) and, more recently, secondary pre-service teachers (Thwaites, Jared, & Rowland, 2011) to help pre-service teachers improve their mathematics teaching. Pre-service teachers and teachers are also encouraged to use the framework as a tool for reflecting on their teaching to enhance their development of MCK (Rowland et al., 2009). The Knowledge Quartet framework focuses on MCK rather than organisation and classroom management. After observing pre-service teachers teaching, 18 codes were identified classifying subject matter knowledge demonstrated during teaching and grouped into four categories: *foundation knowledge, transformation, connection* and *contingency* (Rowland et al., 2009) (see Table 1).

Table 1
The Codes of the Knowledge Quartet (Rowland et al., 2009, p. 29)

Foundation Knowledge	Adheres to textbook	Awareness of purpose
	Concentration on procedures	Identifying errors
	Overt subject knowledge	Theoretical underpinning
	Use of terminology	
Transformation	Choice of examples	Choice of representation
	Demonstration	
Connection	Anticipation of complexity	Decisions about sequencing
	Making connections between procedures	Making connections between concepts
	Recognition of conceptual appropriateness	
Contingency	Deviation from agenda	Responding to children's ideas
	Use of opportunities	

There is a need to develop understanding of how and when pre-service teachers develop their MCK during their teacher education program. Previous studies have reported on pre-service teachers' MCK and PCK when responding to test items near the end of their program (e.g. Beswick & Callingham, 2011; Tatto et al., 2012). Few studies have explored pre-service teachers' MCK during practice teaching experiences. It is also necessary to consider longitudinal studies of teachers' knowledge for teaching.

Details of the Case Study

This case study describes the development of one pre-service teacher's MCK during a four-year, pre-service teacher education program. Merriam (1988) refers to a case study as

"an intensive, holistic description and analysis of a single instance, phenomenon or unit" (p. 21). Consequently, this paper reports on the specific case of Rose one pre-service teacher considering her coursework and practice teaching experiences in primary schools. It is intended that by utilising this approach a deeper understanding may be gained of specific opportunities associated with the development of her MCK.

Rose was one of 17 pre-service teachers who agreed to take part in a four-year longitudinal study of the development of MCK. These pre-service teachers were undertaking a four-year teacher education program, gaining qualifications to teach in primary and secondary schools. All pre-service teachers completed three core primary education mathematics units of study. Some pre-service teachers chose to complete an elective primary mathematics unit, designed to assist them to extend their MCK and gain the knowledge required for passing a Mathematical Competency, Skills and Knowledge (MCSK) test ranging in mathematical understanding from Year 5 to Year 8 (Australian Curriculum Assessment and Reporting Authority [ACARA], 2013).

When analysing pre-service teachers' MCK, multiple data collection methods were used during the qualitative longitudinal study. The pre-service teachers were observed teaching a primary mathematics lesson in second-year and fourth-year. The purpose of the lesson observations, were to gather data related to how pre-service teachers use their MCK when teaching primary mathematics lessons. For example, the researcher focussed on what they said relating to MCK during the lesson; the questions and responses they gave students; the materials and the mathematical terminology used during the lessons. Studying pre-service teachers in a teaching practice setting allowed for a richer understanding of the phenomenon being studied (McMillan, 2004), in this case the development of Rose's MCK.

In addition to the lesson observations, pre-service teachers were interviewed during the second, third and fourth years of their program. These interviews were scheduled for about one hour. During second and fourth year, the interviews were conducted after the lesson observations. There were two main purposes for these semi-structured audio-recorded interviews. The first was to gather additional reflections related to the lesson the researcher had observed. The second was to ask a series of questions as part of the overall study, gathering responses, in particular, that were related to opportunities and influences that enhanced MCK during primary mathematics coursework and teaching practice experiences.

Data collection techniques included lesson observations, interview responses and artefacts such as lesson plans. Field notes were collected during the lesson observation and the researcher did not interact with pre-service teachers or the students during the lesson. The two lessons and three interviews were audio taped and later transcribed for analysis. Data were coded using the four codes of the Knowledge Quartet (Rowland et al., 2009).

Results

Coursework

Rose completed one primary mathematics core unit of study when in first year. During this unit she completed a practice MCSK test, identifying her strengths and weaknesses in MCK. Rose then chose to enrol in an elective primary mathematics unit of study, during summer school and before the beginning of the second year, because she was concerned about her weaknesses in MCK. Then in second-year Rose completed another two core units in primary mathematics teaching.

Rose suggested that focusing on developing her MCK during summer school was useful because she could concentrate on learning the mathematics while she was not completing other units of study or assignments at the same time. In addition to the other three mathematics education units, the elective unit assisted Rose to further develop her *foundation* knowledge.

> Numeracy and Mathematics [the elective unit] was the most helpful in developing my understanding of maths.
>
> I did summer school maths as I did not want to have that and the education maths units at the same time. I thought that was a smart choice. I could just focus on maths and it was on the holidays and I had plenty of time to study. I think that was the most helpful.
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> When I learnt maths initially [during her schooling] I was learning facts and rules, I never understood the meaning behind it. This is why I would always forget the rules. The course that we did in summer … it was about hands on experiences. Doing it themselves physically rather than just doing it on paper.

Rose reported that this coursework experience assisted in developing her MCK and conceptual understanding that was unlike the rule-based learning she remembered from her own secondary school mathematics education.

Practice Teaching Experiences

During the program, pre-service teachers had the opportunity to extend their MCK when practicing their teaching with primary students. For each year of their program, they completed a different school experience. Most practice teaching days occurred once a week during Semester 1 and Semester 2. These experiences also extended pre-service teachers' PCK and curriculum knowledge of primary mathematics teaching.

During the first-year of her program, Rose completed her practice teaching days with Year 1 and Year 2 students. In second-year, this involved Year 5 students and, in fourth-year, she practiced teaching Year 3 and Year 4 students. In third-year, Rose practiced her secondary discipline subject, Humanities, hence not contributing to her MCK. During her interviews, Rose explained that, in first-year, she mainly helped and observed her mentor (the classroom teacher) teaching mathematics lessons. In second-year, Rose taught her first mathematics lesson. When in fourth-year, Rose planned and taught different topics during her weekly school visits. During practice teaching experiences, Rose's mentor teachers would help her plan the lessons and observe her when teaching, providing feedback after the lesson. Rose also explained that, after the lesson, the mentor teachers tended to focus on classroom management rather than providing comments related to Rose's knowledge of mathematics.

Second-Year Observation Lesson

Rose was observed teaching a Year 5 and Year 6 single-sex class of girls. She had planned a measurement lesson that aimed to assist the students to develop their understanding of angles by naming, measuring and constructing angles using protractors. Rose stated the following aims in her lesson plan:

> Use a protractor correctly to measure angles, name the angles with the proper terms, [and] identify angles and estimating angles.

Preparing her second-year lesson provided Rose with an opportunity to think about the purpose of the lesson and the MCK she needed for teaching these concepts. Before teaching a lesson she usually looked up terms on the Internet as part of her planning. Rose

wanted to ensure she had correct understanding of the topic she was about to teach, suggesting her personal beliefs regarding the importance of mathematics.

> I looked up the definitions and I used them in the lesson. I would have had a rough idea of the definitions for the lesson and now I think I know them off by heart…

For this lesson she checked her knowledge of the different angles, listing the key vocabulary in her lesson plan: "angle, degrees, protractor, reflex, revolution". Rose relied on and demonstrated her procedural knowledge when asking the students questions. She mostly asked closed questions as she questioned the students and recorded the names of the angles onto the whiteboard.

> There are certain names for angles can anyone tell me one?... so which one of these is a right angle?... Yes, I am just going to write a definition for a right angle [wrote: an angle measuring 90°]…Can anyone else name one of these angles?

Rose chose not to measure the angles during this part of the lesson with the students, assuming they understood the attribute of the angle size from the diagram. Even though Rose was demonstrating procedural knowledge for naming the angles, her decision not to measure these angles or draw attention to the size of the angle may have confused students and limited their mathematical understanding of this topic. Using appropriate teaching strategies is a key factor when demonstrating *transformation* (Rowland et al., 2009). This example also suggests that Rose should focus on extending her *foundation* knowledge and this should develop her teaching strategies and questioning when naming and labelling angles.

During the lesson Rose assisted the students as they found and measured angles in the classroom. Measuring angles is a difficult concept for students to understand (Van de Walle, Karp, & Bay-Williams, 2012). However, in this situation, Rose was able to *transform* what she knew when assisting students one-on-one when compared to teaching the whole class.

> A protractor has heaps of different lines and it shows you the space, what you want to do is line up the lines and it can make the angle.

Rose designed a worksheet for this lesson using information from the Internet. She had drawn different angles and the students had to estimate the size of the angles, measure them with a protractor then name the type of angle. Designing this worksheet may have assisted Rose to extend her *foundation* knowledge because she had to rely on her MCK when choosing her examples.

When interviewed, Rose demonstrated that she was beginning to make *connections* (Rowland et al., 2009) as she discussed her understanding of the purpose of student activities and developing their mathematical understanding.

> …the teacher needs to be able to not just use the rules but do hands-on activities … in the course you get to understand how the VELS Levels work and so you know what to teach certain kids and how to actually put them into groups and find out what they know…

Also when planning and teaching this lesson Rose was able to extend her *foundation* knowledge by practicing the mathematical terms needed to promote students' understanding.

Fourth-Year Lesson Observation

When in fourth year, Rose's teaching was observed during a lesson on geometry and properties of triangles with a Year 3/4 class. Rose had planned the lesson with her mentor teacher and completed a lesson plan providing an opportunity to revise any MCK required

for this lesson. During the post-lesson interview, Rose also explained that she always checked her MCK before teaching.

> Each week before I go, I talk with my mentor about the maths lesson I will teach the following week. She gives me some ideas then I go home and plan the rest. I try to email it a couple of days before so she can check it but she is usually happy with what I choose to do …

Rose's lesson plan included short definitions of the terms required for the lesson, such as, equilateral, all three sides and angles are equal; reflex angle 180-360 degrees. This most likely assisted revision of her *foundation* knowledge and procedural understanding before the lesson. Rose also checked the names of the different triangles and their properties and mentioned that she relied on her mathematics' learning log from summer school when planning her lessons and checking her MCK.

Rose introduced the lesson by asking the students to brainstorm what was similar and different about a set of laminated triangles she had made. Next they sorted and labelled the triangles into three groups whilst discussing their properties for scalene, isosceles or equilateral triangles. Similarly she used a website dragging and grouping different triangles. Finally the students were given a sheet, which Rose had prepared. Students cut out 14 triangles and sorted them by comparing symmetry, length of sides or size of angles. During this lesson, Rose's reliance on her MCK was revealed by her choice of question types, including open-ended questions to discuss differences and counter examples of triangles demonstrating *foundation* knowledge. She also made *connections* by choosing similar tasks to assist students to identify the differences, similarities and properties of triangles. An appropriate range of examples and representations of triangles illustrated the properties of triangles and extended the students' understandings of these properties, thus providing evidence of *transformation*.

Discussion and Conclusion

A combination of program experiences provided Rose with opportunities that promoted her MCK for primary teaching. Rose was able to identify her need to improve her MCK when in first year and consequently completed an additional, elective unit in primary mathematics teacher education that extended development of her *foundation* knowledge and assisted in passing the MCSK test on her first attempt. *Foundation* knowledge includes a teacher's beliefs about mathematics. During first-year Rose was able to change her beliefs about mathematics because she was developing new understandings of how mathematics is learnt including concentrating on developing conceptual understanding. Having revised her *foundation* knowledge Rose could bring this knowledge to her second-year primary mathematics units of study as well as her practicum teaching experiences. During second year Rose could then begin to shape other categories of the Knowledge Quartet (Rowland et al., 2009).

Further evidence of Rose's development of MCK can be seen in a comparison of her second year and fourth year lesson. In second year, Rose chose closed questions when working with the whole class and in fourth year improved MCK afforded her the opportunity to be more open in her questioning. By fourth year Rose demonstrated the different categories of *foundation* knowledge, *transformation* and making *connections*. She relied on her *foundation* knowledge when introducing mathematical ideas to students, enhanced students' mathematical understanding through her choice of examples, and demonstrated *transformation* and *connections* with her MCK when questioning the

students and considering the sequence of the lesson. In addition, planning with her mentor and her diligent preparation of her lesson plan supported her MCK when teaching.

During practicum experiences Rose had the opportunity to practice her teaching in lower, middle and upper primary classrooms which were complimented by her opportunities to learn MCK during her first and second year coursework experiences. These practice teaching experiences would have assisted Rose develop understanding of the content of the mathematics curriculum by making *connections* of the level of difficulty of different topics because of the depth of MCK she experienced during her three primary practice teaching experiences. Ma (1999) describes "depth of understanding as the capacity to connect a topic with those of greater conceptual power" (p. 124).

Limited data were collected regarding the role of mentor teachers. However, Rose's conversations with her mentor teachers and opportunity to observe her mentors' lessons would have been influential in developing her competence to teach mathematics.

Factors that assisted Rose to develop her MCK during the program were:

- identification of the need to improve her MCK when in first-year;
- responding to this need early in her program by undertaking an elective unit;
- opportunity to practice her teaching across different year levels during her program;
- careful planning and preparation before teaching during her practicum teaching experiences assisted by her mentor teacher.

Debriefing after lessons with her mentor teachers had potential to also assist Rose in developing MCK by focussing feedback on her MCK in action when teaching, but this potential was not realised as they focussed on classroom management skills. These factors should be considered when planning future programs, maximising pre-service teachers' opportunities to learn MCK. Further opportunities to promote Rose's MCK may have been extended by providing an additional fourth year coursework unit of study building on the MCK that Rose learnt during the first two years of her program.

Identifying opportunities and influences that enhance pre-service teachers' MCK is important for improving primary mathematics teacher education. Ensuring that future pre-service teachers entering teacher education identify with the importance of knowing mathematics for primary teaching, seeking opportunities to learn MCK during coursework as well as practice teaching across different year levels may assist with improving the quality of future pre-service teachers' knowledge for teaching mathematics. Roses' opportunities to learn her MCK as well as her desire to want to learn MCK assisted her to extend her MCK during program experiences. Limited conclusions should be drawn from one case study, however the findings from the other longitudinal study pre-service teachers can be compared with these findings and will assist with identifying pre-service teachers' opportunities to promote MCK.

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